Spinmotive force due to electric field with static and uniform magnetization

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A new spinmotive force is predicted in ferromagnets with the spin-orbit coupling. Theory of spinmotive force is extended, in which a time-varying electric field is found to be able to induce a spinmotive force with the static and uniform magnetization. The spinmotive force can be electrically investigated free from the inductive voltage owing to the absence of dynamical magnetization, and can be tuned by the external electric fields. We theoretically demonstrate the spinmotive force in two systems; electric voltage measurement in a single ferromagnet and spin injection from a ferromagnet into an attached nonmagnet.

I. INTRODUCTION

Investigation of mutual interaction between electrons and magnetization is a key subject in the field of spintronics[1]. Spinmotive force (SMF) is one of emerging concepts[2], in which spin current and electric voltage are induced in a ferromagnet due to the exchange coupling between electrons and the magnetization. The SMF provides an important ground for the basic study of the electron-magnetization interaction[2–22], as well as a new concept for spintronic devices[23–25]. The SMF is described by the so-called spin electric field, which accelerates electrons in opposite directions depending on their spin, giving rise to a spin current in the ferromagnet and then an electric voltage. Until recently, two SMFs corresponding to two spin electric fields have been known, which depend on both $\partial_t \boldsymbol{m}$ and $\nabla \boldsymbol{m}$[2–8], where $\boldsymbol{m}$ is the classical unit vector of the magnetization direction, and ∂_t and ∇ represent the derivatives with respect to time and space, respectively. Therefore the appearance of the SMFs is confined in time-varying and spatially-nonuniform magnetization regions, such as moving domain walls[9,10], vortex cores[11], and asymmetrically-petterned films[12]. Recently, it has been pointed out that in a system with the Rashba spin-orbit (SO) coupling there exist additional spin electric fields, which demand only $\partial_t \boldsymbol{m}$[13,14]. The discoveries have enabled us to generate a SMF in time-varying but spatially-uniform magnetic structures, such as a ferromagnetic resonance system.

In this work the theory of SMF is further extended in a system with the general SO coupling beyond the Rashba coupling, in which we predict a new SMF demanding neither $\partial_t \boldsymbol{m}$ nor $\nabla \boldsymbol{m}$; a new spin electric field is found to be proportional to $\boldsymbol{m} \times \partial_t \boldsymbol{E}$, with $\boldsymbol{E}$ a U(1) electric field. Thus, the SMF can be generated by application of time-varying electric fields with the static and uniform magnetization. The SMF has two advantages compared with the other forms of SMFs: (i) The electrical measurement of the SMF is free from the inductive voltage because of no dynamical magnetization. (ii) The SMF can be tuned by the applied electric fields free from the characteristic frequencies inherent in ferromagnets such as the ferromagnetic resonance frequency. We demonstrate the SMF in two systems; electric voltage measurement in a single ferromagnet and spin injection from a ferromagnet into nonmagnet.

II. FORMALISM

In a non-relativistic limit up to the order of $1/c^2$ with c the light speed, Hamiltonian of a conduction electron in a ferromagnetic conductor is written by

$$\mathcal{H} = \frac{\boldsymbol{p}^2}{2m_{\mathrm{e}}} + J_{\mathrm{ex}} \boldsymbol{\sigma} \cdot \boldsymbol{m} - \frac{e\eta_{\mathrm{so}}}{\hbar} \boldsymbol{\sigma} \cdot (\boldsymbol{p} \times \boldsymbol{E}), \tag{1}$$

where m_{e} and $-e$ are the electron's mass and charge, respectively. The second term represents the exchange interaction between the electron spin and the magnetization, with J_{ex} the exchange coupling energy, $\boldsymbol{\sigma}$ the Pauli's matrices. In the third term we introduce a SO interaction, with η_{so} be the SO coupling parameter. In the free electron model $\eta_{\mathrm{so}} = \hbar^2/4m_{\mathrm{e}}^2 c^2$, but in real materials it can be enhanced by several orders of magnitude[26]. The magnetization $\boldsymbol{m}$ and the electric field $\boldsymbol{E}$ are in general dependent on time and space.

To calculate SMFs, let us investigate the equation of motion for the conduction electron. The velocity operator $\boldsymbol{v}$ is given by $\boldsymbol{v} = (1/i\hbar)[\boldsymbol{r}, \mathcal{H}] = \boldsymbol{p}/m_{\mathrm{e}} + (e\eta_{\mathrm{SO}}/\hbar)\boldsymbol{\sigma} \times \boldsymbol{E}$, where the second term in the last line is the so-called anomalous velocity. The "force" $\boldsymbol{\mathcal{F}}$ acting on the electron is given by $\boldsymbol{\mathcal{F}} = (1/i\hbar)[m_{\mathrm{e}}\boldsymbol{v}, \mathcal{H}] + \partial(m_{\mathrm{e}}\boldsymbol{v})/\partial t$. The force $\boldsymbol{\mathcal{F}}$ is a SU(2) operator containing the Pauli matrices $\boldsymbol{\sigma}$ that play as the electron spin operators. To address realistic electron dynamics, in the following the expectation value of the force $\langle \boldsymbol{k} \pm | \boldsymbol{\mathcal{F}} | \boldsymbol{k} \pm \rangle$ is calculated by determining the electron spin dynamics $\langle \boldsymbol{k} \pm | \boldsymbol{\sigma} | \boldsymbol{k} \pm \rangle$, where $|\boldsymbol{k}\pm\rangle$ stands for a one electron state with momentum $\hbar \boldsymbol{k}$ and majority(+) or minority(−) spin.

Assuming is that the electron spin dynamics is described by a Bloch-type equation of motion,

$$\frac{\partial}{\partial t} \langle \boldsymbol{k} \pm | \boldsymbol{\sigma} | \boldsymbol{k} \pm \rangle = -\gamma \langle \boldsymbol{k} \pm | \boldsymbol{\sigma} | \boldsymbol{k} \pm \rangle \times \boldsymbol{m} - \frac{\delta \boldsymbol{m}_\pm}{\tau_{\mathrm{sf}}}, \tag{2}$$

where $\gamma = 2J_{\rm ex}/\hbar$, $\tau_{\rm sf}$ is the relaxation time for the electron spin flip, and $\delta\boldsymbol{m}$ represents a misalignment between the electron spin and the magnetization, which is defined as $\langle \boldsymbol{k} \pm |\boldsymbol{\sigma}|\boldsymbol{k}\pm\rangle = \mp\boldsymbol{m} + \delta\boldsymbol{m}_\pm$. The first term in the right-hand side of Eq. (2) is the Larmor precession around the magnetization $\boldsymbol{m}$, while the second term represents a relaxation process towards $\boldsymbol{m}$ due to scattering by impurities, electrons, and so on, causing a non-adiabaticity in the electron spin dynamics. Here we have assumed that the electron spin dynamics is dominated by the exchange coupling, considering a condition $J_{\rm ex} \gg e\eta_{\rm SO}|\boldsymbol{k}||\boldsymbol{E}|$. The misalignment $\delta\boldsymbol{m}_\pm$ is essential for $\langle \boldsymbol{k} \pm |\boldsymbol{\mathcal{F}}|\boldsymbol{k}\pm\rangle$, although it is in general much small compared to the component $\mp\boldsymbol{m}$. One can easily see that the terms, $\langle \boldsymbol{k} \pm |[m_{\rm e}\boldsymbol{v}, J_{\rm ex}\boldsymbol{\sigma}\cdot\boldsymbol{m}]|\boldsymbol{k}\pm\rangle$ and $\langle \boldsymbol{k} \pm |[(m_{\rm e}e\eta_{\rm SO}/\hbar)\boldsymbol{\sigma}\times\boldsymbol{E}, J_{\rm ex}\boldsymbol{\sigma}\cdot\boldsymbol{m}]|\boldsymbol{k}\pm\rangle$, appearing in the force become zero if $\langle \boldsymbol{k} \pm |\boldsymbol{\sigma}|\boldsymbol{k}\pm\rangle = \mp\boldsymbol{m}$.

Let us decompose $\delta\boldsymbol{m}_\pm$ into two directions perpendicular to the magnetization as $\delta\boldsymbol{m}_\pm = X_\pm\boldsymbol{m}\times d\boldsymbol{m}/dt + Y_\pm d\boldsymbol{m}/dt$, where $X_\pm$ and $Y_\pm$ are spin-dependent constants, and d/dt is the Lagrange derivative as $d/dt = \partial/\partial t + \langle \boldsymbol{k} \pm |\boldsymbol{v}|\boldsymbol{k}\pm\rangle\cdot\nabla$. By substituting the expression of $\langle \boldsymbol{k} \pm |\boldsymbol{\sigma}|\boldsymbol{k}\pm\rangle$ into Eq. (2) and comparing the left-hand and right-hand sides, the explicit forms of $X_\pm$ and $Y_\pm$ are obtained. In the process, the term $\partial\delta\boldsymbol{m}_\pm/\partial t$, which is the order of $\partial^2\boldsymbol{m}/\partial t^2$, is discarded. The electron spin is in the end expressed in terms of the magnetization as[5]

$$\langle \boldsymbol{k}\pm|\boldsymbol{\sigma}|\boldsymbol{k}\pm\rangle = \mp\left[\boldsymbol{m} - \frac{\hbar}{2J_{\rm ex}}\left(\boldsymbol{m}\times\frac{d\boldsymbol{m}}{dt} + \frac{\hbar}{2J_{\rm ex}\tau_{\rm sf}}\frac{d\boldsymbol{m}}{dt}\right)\right]. \tag{3}$$

By using Eq. (3), the expectation value of the force that acts on the conduction electron is written by

$$\langle \boldsymbol{k} \pm |\boldsymbol{\mathcal{F}}|\boldsymbol{k}\pm\rangle = -e\boldsymbol{\mathcal{E}}_\pm, \tag{4}$$

where $\boldsymbol{\mathcal{E}}_\pm$ is the spin electric field that is given by

$$\begin{aligned}\boldsymbol{\mathcal{E}}_\pm = &\pm\frac{\hbar}{2e}\boldsymbol{m}\times\frac{\partial\boldsymbol{m}}{\partial t}\cdot\nabla\boldsymbol{m} \pm \frac{\hbar}{2e}\frac{\hbar}{2J_{\rm ex}\tau_{\rm sf}}\frac{\partial\boldsymbol{m}}{\partial t}\cdot\nabla\boldsymbol{m}\\ &\pm\frac{m_{\rm e}\eta_{\rm SO}}{\hbar}\frac{\partial}{\partial t}(\boldsymbol{m}\times\boldsymbol{E})\\ &\mp\frac{m_{\rm e}\eta_{\rm SO}}{\hbar}\frac{\hbar}{2J_{\rm ex}\tau_{\rm sf}}\left(\boldsymbol{m}\times\frac{\partial\boldsymbol{m}}{\partial t}\right)\times\boldsymbol{E}.\end{aligned} \tag{5}$$

In Eq. (4), the velocity-dependent terms are discarded, which include the spin magnetic fields causing the anomalous Hall effect due to the scalar spin chirality[27,28] and due to the SO interaction[29]. We now consider an open circuit condition where the ensemble average of $\langle \boldsymbol{k} \pm |\boldsymbol{v}|\boldsymbol{k}\pm\rangle$ is zero, and focus on the effects of the spin electric field $\boldsymbol{\mathcal{E}}_\pm$.

The spin electric field (5) accelerates the electrons with majority and minority spins in the opposite directions each other, giving rise to a spin current that is accompanied by an electric voltage. The first two terms in Eq. (5) purely come from the exchange interaction. The first term has been known as the origin of the conventional SMF[2–5]. The second term reflects the non-adiabaticity in the electron spin dynamics[6–8], which goes to zero in the adiabatic limit $J_{\rm ex}\tau_{\rm sf}\to\infty$. Since these two terms depend on $\partial\boldsymbol{m}/\partial t$ and $\nabla\boldsymbol{m}$, appearance of the SMFs due to these spin electric fields are spatially confined in time-varying and spatially-nonuniform magnetization regions. The last two terms in Eq. (5), which do not include $\nabla\boldsymbol{m}$, appear due to the combination of the exchange and SO interactions. The fourth term, reflecting the non-adiabatic dynamics of the electron spin, was recently derived in the Rashba SO coupling systems based on the perturbative calculation[14]. We found a new spin electric field, which is contained in the third term. In Ref. 13, the spin electric field proportional to $\partial\boldsymbol{m}/\partial t\times\boldsymbol{E}$, which is included in the third term in Eq. (5), was found in the Rasahba SO coupling systems, where the electric field $\boldsymbol{E}$ due to the inversion symmetry is assumed to be static. Starting from the general Hamiltonian (1) where $\boldsymbol{E}$ can vary in time, we found that there appears a new spin electric field proportional to $\boldsymbol{m}\times\partial\boldsymbol{E}/\partial t$.

It should be noted that since the SMF can be induced with the static and uniform magnetization, we can investigate the SMF electrically in detail under no disturbance arising from the inductive voltage, in contrast to the other SMFs. In addition, the SMF is tuned by the applied electric fields with variable frequencies, whereas the other SMFs' time-dependence is restricted by the characteristics of the magnetization dynamics. In the following, we propose two systems to demonstrate the predicted SMF.

III. VOLTAGE MEASUREMENT

First, let us consider a thin film of ferromagnetic conductor, which has a static and uniform magnetic structure $\boldsymbol{m}(\boldsymbol{r},t) = \hat{\boldsymbol{y}}$ and is subjected to a space-independent ac electric field $\boldsymbol{E}(\boldsymbol{r},t) = E_0\sin\omega t\hat{\boldsymbol{z}}$, with E_0 and ω the amplitude and the angular frequency of the electric field, respectively, and $\hat{\boldsymbol{i}}$ the unit vector along the i axis ($i = x$, y, or z). Here $\hat{\boldsymbol{z}}$ axis is set to be normal to the film plane (see Fig. 1. a). In this condition, Eq. (5) is reduced to

$$\boldsymbol{\mathcal{E}}_\pm = \pm\frac{m_{\rm e}\eta_{\rm SO}}{\hbar}E_0\omega\cos\omega t\hat{\boldsymbol{x}}. \tag{6}$$

In this section, the electric voltage due to the spin electric field (6) is investigated.

The difference in the electric conductivities of the majority and minority electrons, $\sigma_F^\uparrow$ and $\sigma_F^\downarrow$, respectively, results in the charge current $\boldsymbol{j}_{\rm c}(t)$, which is given by

$$\boldsymbol{j}_{\rm c}(t) = \sigma_F^\uparrow\boldsymbol{\mathcal{E}}_+ + \sigma_F^\downarrow\boldsymbol{\mathcal{E}}_- = \frac{P\sigma_F m_{\rm e}\eta_{\rm SO}}{\hbar}E_0\omega\cos\omega t\hat{\boldsymbol{x}}. \tag{7}$$

Here P is the spin polarizarion defined as $P = (\sigma_F^\uparrow - \sigma_F^\downarrow)/(\sigma_F^\uparrow + \sigma_F^\downarrow)$, and $\sigma_F = \sigma_F^\uparrow + \sigma_F^\downarrow$. Exactly speaking, the complex admittances should be used instead of the conductivities as we are considering the ac charge current. Although, for simplicity we here consider a condition where the reactance of the circuit is small enough

so that the admittances are well approximated by the conductivities.

In the open circuit condition, the charge current (7) is cancelled by the electric charge rearrangement, giving rise to an electric potential distribution $\phi(x,t)$ so that $\boldsymbol{j}_c(t) - \sigma_F \partial\phi(x,t)/\partial x = 0$. The electric voltage V appearing between the sample edges, where $x = -L$ and 0, is provided by

$$V = \int_{-L}^{0} dx \frac{\partial \phi(x,t)}{\partial x} = \frac{P m_{\mathrm{e}} \eta_{\mathrm{SO}} L}{\hbar} E_0 \omega \cos \omega t. \tag{8}$$

The amplitude of the voltage can be tuned by the distance between the electrodes L and the angular frequency of the electric field ω. Notice that V and $\boldsymbol{E}$ vary in time with the same angular frequency ω, but their phases are different by $\pi/2$ since the spin electric field is proportional to the time derivative of $\boldsymbol{E}$; $V \propto \cos\omega t$ while $E_z \propto \sin\omega t$, indicating that one can readily distinguish the SMF signal from the possible anomalous Hall voltage, which is proportional to $\boldsymbol{E}$ itself. No inductive voltage appears in the present system because there is no dynamical magnetization.

In Fig. 1 b and c, the time evolution of Eq. (8) is shown together with that of the applied electric field. The amplitude of V is ~ 30 μV, adopting the typical parameters in a thin film of ferromagnetic metals: $m_{\mathrm{e}} = 9.1 \times 10^{-31}$ kg, $\eta_{\mathrm{so}} = 10^{-21}$ m^2, $P = 0.5$, $E_0 = 10^8$ V/m, $\omega = 2\pi \times 10^8$ s^{-1}, and $L = 100$ μm.

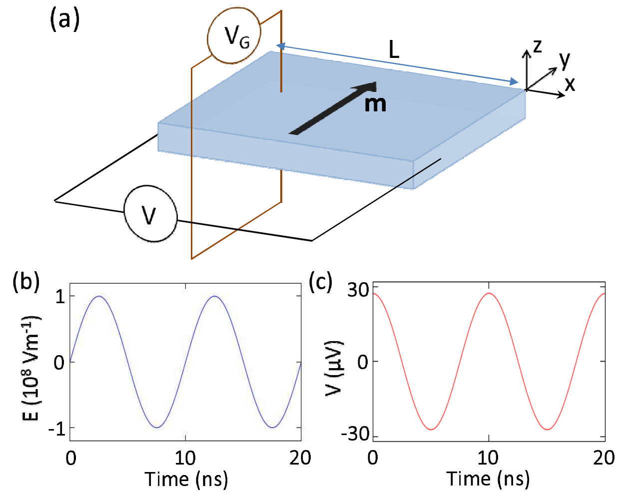

FIG. 1. (a) Measurement schem of the electric voltage V due to the SMF with the static and uniform magnetization $\boldsymbol{m}$. The electric field varing sinusoidally in time is applied by the gate voltage V_G. The electric voltage appears in perpendicular to both the time-derivative of the electric field and the magnetization. (b)-(c) Time evolution of a certain applied electric field and the corresponding electric voltage V. The ac electric field gives rise to the ac electric voltage with the same frequency and the $\pi/2$ phase shift. The amplitude of V is proportional to the angular frequency of the electric field and the distance between the electrodes L.

IV. SPIN INJECTION

Next, we investigate a spin injection method by using the spin electric field (6). Let us consider a nonmagnetic metal (N) attached to the ferromaget (F), which has the in-plane magnetization and is subjected to the sinusoidally-varying electric field as before (Fig. 2 a). In F layer, the spin electric field (6) induces not only the charge current (7) but also the spin current

$$\boldsymbol{j}_{\mathrm{s}} = -\left(\sigma_F^{\uparrow} \boldsymbol{\mathcal{E}}_+ - \sigma_F^{\downarrow} \boldsymbol{\mathcal{E}}_-\right) = -\frac{\sigma_F m_{\mathrm{e}} \eta_{\mathrm{SO}}}{\hbar} E_0 \omega \cos \omega t \hat{\boldsymbol{x}}, \tag{9}$$

giving rise to a spin accumulation at the ends of F, which diffuses into N and decays within the spin diffusion length (Fig. 2 b). The injected spin current into N, $\boldsymbol{j}_{\mathrm{s}}^N$, is calculated below.

The spin accumulations in F and N, $\mu_{F(N)}^{\uparrow} - \mu_{F(N)}^{\downarrow}$, with $\mu_{F(N)}^{\uparrow(\downarrow)}$ the electrochemical potential for a electron with majority (minority) spin in F (N), obeys the diffusion equation[1,7]

$$\nabla^2 (\mu_{F(N)}^{\uparrow} - \mu_{F(N)}^{\downarrow}) = \frac{1}{\lambda_{F(N)}^2} (\mu_{F(N)}^{\uparrow} - \mu_{F(N)}^{\downarrow}) - 2e \nabla \cdot \boldsymbol{\mathcal{E}}_+, \tag{10}$$

where $\lambda_{F(N)}$ is the spin diffusion length in F (N). By substituting the spin electric field (6), which appears only in F, into Eq. (10), the forms of the solutions are;

$$\mu_F^{\uparrow} - \mu_F^{\downarrow} = A_{F1} e^{x/\lambda_F} - A_{F2} e^{(x+L)/\lambda_F}, \tag{11}$$

$$\mu_N^{\uparrow} - \mu_N^{\downarrow} = A_N e^{-x/\lambda_N}, \tag{12}$$

where the origin of the x axis is located at the F/N interface. Here we assume that N is much wider than λ_N in the x direction, so that the spin accumulation at another end of N can be neglected. The coefficients A_{F1}, A_{F2}, and A_N are determined from the boundary conditions for the electrochemical potentials and the charge and spin currents: $\mu_F^{\uparrow(\downarrow)}(0,t) = \mu_N^{\uparrow(\downarrow)}(0,t)$, $\boldsymbol{j}_{\mathrm{s}}(-L,t) = 0$, $\boldsymbol{j}_{\mathrm{s}}(0,t) = \boldsymbol{j}_{\mathrm{s}}^N(0,t)$, and the charge current is zero both in F and N because of the open circuit condition. Thus we obtain

$$A_{F1} = A_N = -\frac{1}{1+\alpha} 2e\lambda_F \frac{m_{\mathrm{e}} \eta_{\mathrm{so}}}{\hbar} E_0 \omega \cos \omega t, \tag{13}$$

$$A_{F2} = 2e\lambda_F \frac{m_{\mathrm{e}} \eta_{\mathrm{so}}}{\hbar} E_0 \omega \cos \omega t, \tag{14}$$

where α is a dimensionless parameter defined as

$$\alpha = \frac{\lambda_F \sigma_N}{\lambda_N \sigma_F (1-P^2)}, \tag{15}$$

with σ_N the electric conductivity of N. The spin current in N is given by

$$\boldsymbol{j}_{\mathrm{s}}^N = -\frac{\sigma^N}{e} \nabla \left(\mu_N^{\uparrow} - \mu_N^{\downarrow}\right), \tag{16}$$

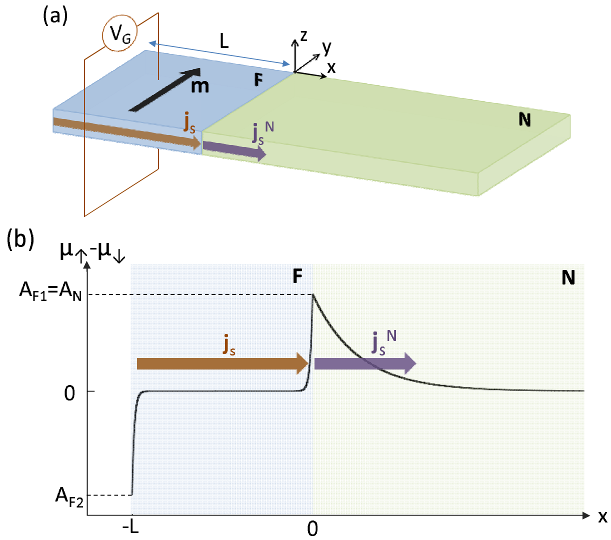

FIG. 2. (a) Schematic of the spin injection from a ferromagnet (F) into a nonmagnet (N). The spin current $\boldsymbol{j}_{\rm s}$ induced in F due to the spin electric field (6) gives rise to a spin accumulation at the F/N interface, which decays in N as the diffusive spin current $\boldsymbol{j}_{\rm s}^N$ within the spin diffusion length. (b) Spatial dependence of the spin accumulation. The spatial gradient of the spin accumulation gives the injected spin current $\boldsymbol{j}_{\rm s}^N$. The coefficients A_{F_1}, A_{F_2} and A_N is determined by solving the diffusion equation (10) for F and N layers.

which oscillates in time with the angular frequency ω. Adopting the same parameters as before and $\lambda_N = 1$ μm, $\lambda_F = 10$ nm, and $\sigma_F = \sigma_N = (1\ \mu\Omega\cdot\text{cm})^{-1}$, the amplitude of $\boldsymbol{j}_{\rm s}(0,t)$ at the F/N interface is $\sim 5\times 10^5$ A/m^2.

V. CONCLUSION

In conclusion, the theory of spinmotive force has been extended in a system with the spin-orbit coupling, and a new spinmotive force was derived, which can be induced by time-varying electric fields with the static and uniform magnetization. The spinmotive force has two advantages compared with the other SMFs: (i) The electrical measurement of the spinmotive force is free from the inductive voltage. (ii) The spinmotive force can be tuned by the applied electric fields free from the characteristic frequencies inherent in ferromagnets such as the ferromagnetic resonance frequency. We have demonstrated the spinmotive force in two systems; electric voltage measurement in a single ferromagnet and spin injection from a ferromagnet into a nonmagnet.

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